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CONTRIBUTIONS TO THE STUDY OF NONLINEAR PHENOMENA IN QUANTUM MECHANICS

Summary of doctorate thesis

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CHAPTER 2

CONTRIBUTIONS TO THE STUDY CASIMIR EFFECT BASED ON NONLINEAR DYNAMICS ELEMENTS

2.1 Navier-Stokes equations in Scale Relativity Theory

For viscous compressible fluids, Navier-Stokes equations

$$\rho \frac{D\mathbf{v}}{Dt} = \rho\mathbf{X} - \nabla p + \mu\nabla^2\mathbf{v} + \frac{\mu}{3}\nabla\left(\nabla\cdot\mathbf{v}\right) \tag{2.25}$$

together with the equation of continuity

$$\frac{D\rho}{Dt} + \rho\nabla\cdot\mathbf{v} = 0, \tag{2.26}$$

where ρ is the density, $\boldsymbol{v}$ the velocity of the fluid, $\boldsymbol{X}$ the body force, p the pressure, μ the shear viscosity and $D/Dt \equiv d/dt + \mathbf{v}\cdot\nabla$ the Eulerian derivative, apply to *Newtonian fluids.* Substituting the operator *d/dt* with of the fractal $\hat{d}/dt$ and separating the real and imaginary parts, in a stationary isotropic case, taking the body force $\boldsymbol{X} = 0$ and $\nabla\boldsymbol{U} = 0$ become:

$$\boldsymbol{V}\cdot\nabla\boldsymbol{V} = -\frac{\nabla p}{\rho} + \upsilon\,\nabla^2\boldsymbol{V} \qquad \boldsymbol{U}\cdot\nabla\nabla + \mathscr{D}\nabla^2\boldsymbol{V} = 0$$
$$\boldsymbol{V}\cdot\nabla\rho + \rho\nabla\cdot\boldsymbol{V} = 0 \qquad \boldsymbol{U}\cdot\nabla\rho + \mathscr{D}\nabla^2\rho = 0 \tag{2.27a,b 2.28 a,b}$$

where $\hat{\boldsymbol{V}} = \boldsymbol{V} - i\boldsymbol{U}$ is the complex velocity (we have identified the real velocity $\boldsymbol{V}$ with $\boldsymbol{v}$, the instantaneous velocity of the particle) $\upsilon = \mu/\rho$ the kinematic viscosity and $D = \hbar/2m$ defines the amplitude of the fractal fluctuations.

If we compare it with Navier-Stokes equation (2.27a), where there are no pressure gradients, we can see the first term of (2.27b) gives the rate at which ***V*** is transported through a 'fluid' by means of the motion of 'fluid' particles with the velocity ***U***; the second term gives the diffusion of ***V***, but $\mathscr{D}$ plays the role of the "cinematic viscosity" of the "fluid". If we consider the flow of ***V*** induced by a uniform translation motion of a plane spaced a distance *Y* above a stationary parallel plane, and if the 'fluid' velocity increases from zero (at the stationary plane) to *U* (at the moving plane) like in the case of simple Couette flow, or simple shear flow, then $\text{rate of shear deformation} = \frac{dV}{dy} = \frac{U}{Y}$.

When such 'fluids' flow, it have found at reasonable speeds, the viscous effects appear only in thin layers on the surface of objects or surfaces over which the 'fluid' flows. That is, if one continues the analogy, and questions how is ***V*** transported by the motion of 'fluid' particles with the velocity ***U***, in equation (2.27 b), one can assume that the mechanism of transfer of ***V*** from one particle of 'fluid' to another is achieved over small distances (in thin layers, as stated above).

We study an important case, of the one-dimensional flow along the *Ox* axis $\mathbf{V} = \zeta(x)\mathbf{k}$. Consequently, (2.27 b) reduces to the scalar equation $\zeta''(x) + K^2(x)\zeta(x) = 0$

which is the time independent Schrödinger equation. In this equation $K^2(x) = 1/(\Lambda \mathscr{D})\, U(x)$ with Λ and D having the significance of a small elementary distance and of the 'cinematic viscosity' (or amplitude of the fractal fluctuations), respectively, and $U(x)$ is the velocity of the "Newtonian fluid" [38].

For $\mathscr{D} = \hbar/2m$ and small distances of the order of Compton length, $\Lambda = \hbar/mc$ [39], equation (2.33) is solved using the WKBJ approximation method [40-42]. We obtained for different shapes of functions and quantization conditions, and we demonstrated that if the "potential" well have both walls infinitely steep, like is the case in the Casimir geometry, the quantization rule is given [40-42] $\int_{x_1}^{x_2} k dx' = n\pi,$ where k is the wave number and $n = 1, 2, 3, ...$

2.2 Casimir Type Force in Scale Relativity Theory

The Casimir effect are described by quantum field theories, considering that all of the various fundamental fields, and in particular, the electromagnetic field, must be quantized at each and every point in space.

In our model, let us consider the vacuum, as a non-differentiable, Newtonian, *2D* non-coherent quantum fluid whose entities (quasi-particles)

assimilated to vortex-type objects [43] (see Fig. 2.3) described by the wave function Ψ [44,45] $\Psi = cn(\underline{u};k)$ with

$$\underline{u} = \frac{K}{a}\underline{z}, \quad \underline{z} = x + iy, \quad \frac{K}{a} = \frac{K'}{b}, K = \int_0^{\pi/2}\left(1 - k^2\sin^2\varphi\right)^{-1/2} d\varphi,$$
$$K' = \int_0^{\pi/2}\left(1 - k'^2\sin^2\varphi\right)^{-1/2} d\varphi, \quad k^2 + k'^2 = 1 \qquad (2.\ 37\ a\text{-}f)$$

and *K, K'* complete elliptic integrals of the first kind of modulus *k* [46], form a vortex lattice of constants *a, b*. Applying in the complex plane [47], the formalism developed in [38] by means of the relation $\Psi = e^{F(z)/\Gamma} = cn(\underline{u};k)$ one introduces the complex potential

$$F(\underline{z}) = G(x,y) + iH(x,y) = \Gamma \ln\left[cn(\underline{u};k)\right] \qquad (2.38)$$

with Γ the vortex constant. In our case $\Gamma = c\Lambda = \hbar/m$ [43-45], with Λ Compton length, the interaction scale being specified through Γ's value.

Based on the complex potential (2.38), one defines the complex velocity field of the non-coherent quantum fluid, through the relation

$$v_x - iv_y = \frac{dF(\underline{z})}{d\underline{z}} = -\frac{\Gamma K}{a}\frac{sn(\underline{u};k)dn(\underline{u};k)}{cn(\underline{u};k)} \qquad (2.39)$$

Having in view that $cn(\underline{u} + \underline{\Omega}) = cn(\underline{u})$, where $\underline{\Omega} = 2(2m+1)K + 2niK'$ and *m,n* = ±1, ±2 ..., for $k \to 0$ and $k' \to 1$ limits, respectively, the quantum

fluid, initially non-coherent (the amplitudes and phases of quantum fluid entities are independent) becomes coherent (the amplitudes and phases of quantum fluid entities are correlated [49]). In such a context, the distribution curves of the equipotential curves give in Fig. 2.4 a,b , for $k^2 \to 0$ şi $k^2 \to 1$, it results that the coherence of the quantum fluid reduces to its ordering in vortex streets.

Now, writing the Navier-Stokes equation (2.27a) and the equation of continuity (2.28a) in scale relativity theory for constant density (incompressible fluids) in two dimensions, one gets

$$\begin{aligned}
\frac{\partial p}{\partial x} &= \rho \mathcal{D}\left(\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2}\right) - \rho\left(v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y}\right) \\
\frac{\partial p}{\partial y} &= \rho \mathcal{D}\left(\frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2}\right) - \rho\left(v_x \frac{\partial v_y}{\partial x} + v_y \frac{\partial v_y}{\partial y}\right) \qquad (2.42a,b\ 2.43) \\
\frac{\partial v_x}{\partial x} &+ \frac{\partial v_y}{\partial y} = 0
\end{aligned}$$

where the shear viscosity υ is replaced by $\mathcal{D}$ since we are dealing here with a non-differentiable quantum fluid. In other words, non-differentiability and coherence of the quantum fluid due to constraints, generate pressure along the *Ox* and *Oy* axis.

Let us study the case of a Casimir cavity, rectangular plates with sides d_1, d. The plates induce constraints along both Ox and Oy axis, so that the vortex streets are formed along these directions. Mathematical, operate simultaneously degenerations of the potential $cn(u)$. It follows that every point (x, y) there is a pressure formed of the two constraints, pressure acting on the sides of the rectangular enclosure:

$$\frac{p_{rect}}{p_0'} = -4nr^2 \frac{arctg\left[tg\left(\frac{m\pi^2}{4}\right)th\left(\frac{n\pi^2}{2}r\right)\right]}{th\left(n\pi^2 r\right)} + 4nr \frac{arctg\left[tg\left(\frac{n\pi^2}{2}\right)th\left(\frac{m\pi^2}{4}\frac{1}{r}\right)\right]}{tg\left(n\pi^2\right)} +$$
$$+ 2nr^2 tg\left(\frac{m\pi^2}{4}\right) - 2nr\, th\left(\frac{m\pi^2}{4}\frac{1}{r}\right) -$$
$$- 4m \frac{arctg\left[tg\left(\frac{n\pi^2}{4}\right)th\left(\frac{m\pi^2}{2}\frac{1}{r}\right)\right]}{th\left(m\pi^2\frac{1}{r}\right)} + 4mr \frac{arctg\left[tg\left(\frac{m\pi^2}{2}\right)th\left(\frac{n\pi^2}{4}r\right)\right]}{tg\left(m\pi^2\right)} +$$
$$+ 2m\, tg\left(\frac{n\pi^2}{4}\right) - 2mr\, th\left(\frac{n\pi^2}{4}r\right) \tag{2.56}$$

Plots of (2.56) for various values of parameters m, n = 1, 2, ... and r are depicted in Fig. 2.5a,b. One can notice that if the two parameters m and n have close values, the force acting on the Casimir rectangle is always negative and decreases exponentially for increasing r. For parameters m and n (1,5 and 5,1, very asymmetric) the force has negative and positive domains (see Fig. 2.5b) and increases exponentially for increasing r (Figure 2.5 b). Moreover, the positive and negative domains are obtained

by solving (2.56) for $m = 5$, $n = 5$ finds $p_{rect} < 0$ for $0.45753 \leq r \leq 2.18565$ and $p_{rect} > 0$ for $r > 2.18565$ and $r < 0.45753$. This result is in agreement with the calculus of regularization using the Abel-Plana formula where $E < 0$ for $0.36537 \leq L/l \leq 2.73686$ and $E > 0$ for $L/l > 2.73686$ and $L/l < 0.36537$ [52].

CHAPTER 3

ATOMIC MODELS USING NONLINEAR DYNAMICS

3.1 The unified model of the atom using fractal approximation of motion

3.1.1 Excited states of atoms

Let us consider the interaction of a beam of charged particles with a combined field (monochromatic electromagnetic field and constant external magnetic field). The mathematical model of this dynamical system is based, according to [66-68] on Maxwell equations and relativistic equation of motion for a single particle of the beam.

To obtain the values of parameters which define an efficient mechanism of acceleration (gun effect), we consider that the motion of a single particle of the beam occurs in the electromagnetic field generated by the remaining particles of the beam. Thus the initial system with a self consistent field is transformed in a system with external fields. In these conditions, we will consider the motion of a charged particle in a constant

external magnetic field and in the presence of a transversal electromagnetic field.

We obtain the following dimensionless equations which describe the nonlinear dynamics of the model [66-68]:

$$\frac{dX}{dT}=\frac{P_x}{\sqrt{1+P_x^2+P_y^2}},\frac{dP_x}{dT}=\Omega_c\left[\beta\cos\left(X-T\right)+1\right]P_y$$

$$\frac{dP_y}{dT}=-\Omega_c\left[\beta\cos\left(X-T\right)+1\right]P_x+H\cos\left(X-T\right),\beta=\frac{H}{\Omega_B}=\frac{\left|\Omega_b\right|}{\Omega_B} \qquad (3.82\text{ a-c})$$

significance of parameters are given in thesis.

Analytical solutions for the system (3.82) are difficult to obtain. This is why we shall numerically integrate them. The numerical solutions of these equations and their correspondences with the dynamics of the system have been obtained by applying the fifth-order Runge-Kutta algorithm with an adaptive step error [66-68].

3.1.1.2 Lyapunov exponents

In Figure 3.5 analyzes the chaotic zones by calculating the Lyapunov exponents. The darker areas represent the zones with higher Lyapunov exponent, i.e. the chaotic zones. We can thus confirm the fact that in order to obtain extensive chaotic regimes we must have $H>2.5$. Thus, there are three distinct chaotic "islands" (1 for $H>2.5$ respectively 2 and 3 for $4<H<5$).

3.1.1.3 The time series, Poincaré sections, the phase space

It is difficult to directly observe an evolution towards chaos because the form of circular trajectories does not change significantly (only in certain specific cases the particle returns). We present the particle behavior in the combined field using Lyapunov diagram given in Figure 3.5. Distinguish the following sequences:

i) For small amplitudes of the electromagnetic field, e.g. $H = 0.05$, the particle motion is complex but retains a regular character (see Figs. 3.11 a-g see thesis).

ii) The onset of fractalisation (by means of stochasticity) is observed once H exceeds 0.5 (see Figs. 3.12 a-g, see thesis).

iii) When $H = 1.5$, a gun effect is initiated (see Figs. 3.13 a-g, see thesis).

iv) An extensive chaotic regime is obtained for $H = 2.5$ (see Figs.3.14, a-g see thesis).

v) A chaotic gun effect erupts for $H = 3.5$ (see Figs. 3.15 a-g, see thesis).

vi) A multi-gun effect results for $H = 4.5$ (see Figs. 3.16 a-g, see thesis).

Each sequence was characterized by time series (X,T), $(P_x,T),(P_y,T)$, Poincaré section (P_x,P_y) and phase space (X,P_x), $(X,P_y),(X,P_x,P_y)$.

3.1.1.4 Bifurcation diagrams

The bifurcation diagrams confirm this scenario of transition to chaos resonances overlapping (see Figs. 17).

Partially chaotic zones are zones which points vertical distribution is chaotic transition zones. Transition to chaotic areas is not achieved through successive bifurcations but by resonances overlapping.

The model can describe the transitional stationary states of the atom accepting that "dynamics" system charged particle-combined electromagnetic field can be treated as "dynamics" electron-nucleus system.

3.1.2 Stationary orbits of the atom

Previous analysis states that electrons move around the nucleus takes place on fractal curves. This it may apply relativity formalism [71-76]. For the irotational movements in the external scalar field it results the hydrodynamic fractal system

$$\begin{aligned} m_0\left(\partial_t \mathbf{V} + \mathbf{V}\cdot\nabla\mathbf{V}\right) &= -\nabla Q + \mathbf{U} \\ \partial_t \rho + \nabla\cdot\left(\rho\mathbf{V}\right) &= 0 \end{aligned} \qquad (3.91\ a, b)$$

with Q the fractal potential

$$Q = -\frac{\hbar^2}{2m_0}\frac{\Delta\sqrt{\rho}}{\sqrt{\rho}} = -\frac{m_0\mathbf{U}^2}{2} - \frac{\hbar}{2}\nabla\cdot\mathbf{U} \qquad (3.92)$$

The presence of the scalar potential of the complex speed field implies by irotational movements the followings:

i) at differentiable scale, the real speed field is responsible for the stationary orbits

$$\oint m_0\mathbf{V}\cdot d\mathbf{r} = \hbar m\oint\frac{\mathbf{e}_\Phi d\mathbf{r}}{r\sin\delta} = \hbar m\int_0^{2\pi} d\Phi = hm \qquad (3.98)$$

ii) at non-differentiable scale, the fractal speed field (the imaginary part of the complex speed field) is responsible for the fractal potential

$$Q_{nm}(r) = -\frac{\hbar^2}{2m_0}\left(\frac{1}{n^2a^2} - \frac{2}{ar} + \frac{m^2}{r^2\sin^2\delta}\right) \qquad (3.100)$$

This result was obtained by means of the recurrence relation for the associated Laguerre and Legendre polynomials [10, 11]. The fractal potential leads to the observable energy of the system

$$E_n = \frac{1}{2} m_0 v^2 + U(r) + Q_{nm}(r) = -\frac{\hbar^2}{2m_0} \frac{1}{a^2 n^2} \tag{3.101}$$

where a is the range of the first Bohr orbit

CHAPTER 4

USING NONLINEAR DYNAMICS IN THE STUDY OF INTERACTIONS ON A MICROSCOPIC SCALE

4.1 Transport equation

Considering the complexity of drug release processes from the matrix polymer is substituted by fractals, using the [104], we obtain the transport equation for a quantity *Q* in the form

$$\frac{\hat{\partial} Q}{\partial t} = \frac{\partial Q}{\partial t} + \left(\hat{\boldsymbol{V}} \cdot \nabla\right) Q - i\mathcal{D}\left(dt\right)^{(2/D_F)-1} \Delta Q + \frac{\sqrt{2}}{3} \mathcal{D}^{3/2} \left(dt\right)^{(3/D_F)-1} \nabla^3 Q = 0 \tag{4.55}$$

where $\hat{\boldsymbol{V}}$ is complex speed field. This means that at any point of a fractal path, the local temporal $\partial_t Q$, the non-linearly, $\left(\hat{\boldsymbol{V}} \cdot \nabla\right) Q$, the dissipation, ΔQ, and the dispersion, $\nabla^3 Q$, make their balance.

4.2. The dissipative approximation in the drug release procces.

4.2.1 The generalized diffusion equation

The dynamics of the fractal released drug concentration field at both mesoscopic and nano scale are described by the equation [105]:

$$\frac{\hat{\partial} Q}{\partial t} = \frac{\partial Q}{\partial t} + \left(\hat{\mathbf{V}} \cdot \nabla\right) Q - i\mathcal{D}(dt)^{(2/D_F)-1} \Delta Q = 0 \tag{4.56}$$

The diffusion equation generalized results from (4.59) imposing the following restrictions:

i) the "diffusion" path are fractal curves with fractal dimension $D_F \neq 2$;

ii) the time resolution, δt, is identified with the differential element dt, i.e. the substitution principle can be applied;

iii) the movements at differentiable and non-differentiable scales are "synchronous"

(the same drug release mechanisms at fractal scale manifests, also, at differentiable scale), i.e. $\boldsymbol{V} = \boldsymbol{U}$.

Then, the equation (4.59) can be written $\frac{\partial Q}{\partial t} = \mathcal{D}(dt)^{(2/D_F)-1} \Delta Q$,

which we call the generalized diffusion equation. Considering that the relative variation of released drug concentrations, time dependent, is defined as [105]: $T(t) = (Q_\infty - Q)/Q_\infty$, where Q and Q_∞ are cumulative concentrations of drug released at time t and respectively, infinite time [105], satisfy the generalized diffusion equation using separating variables method [105], resulting formula

$$\frac{Q}{Q_\infty} = 1 - \exp\left[- \frac{l^2 \mathcal{D}}{\Gamma\left(\frac{2}{D_F} + 1 \right)} t^{\frac{2}{D_F}} \right] \tag{4.68}$$

Here, with the substitutions

$$a = \left(\frac{m\pi}{L} \right)^2 \frac{\mathcal{D}}{\Gamma\left(\frac{2}{D_F} + 1 \right)}, \quad b = \frac{2}{D_F} \tag{4.70 a, b}$$

Weibull's is obtained

$$\frac{Q}{Q_\infty} = 1 - \exp(-at^b) \tag{4.69}$$

Constant *b* depends only on the dimension fractal "curves" on which drug release mechanism take place, while constant *a* depends both on the fractal dimension and diffusion order m.

For different values of *a* and/or *b* parameters, the following cases can be distinguished.

i) For *b=1*, that implies $D_F = 2$. Also, this condition implies the next considerations:

- the diffusion paths are the fractal curves of Peano's type ($D_F = 2$).

- the movements at differentiable and non-differentiable scales are synchronous, $\boldsymbol{V} = \boldsymbol{U}$;

- the structure coefficient $\mathcal{D}$ is identified with the diffusion coefficient, i.e. $\mathcal{D} \equiv D$.

ii) If in the relation (4.69) we consider the restriction on time $t \langle\langle 1/a^b$, with $a \langle 1$, this can be reduced to a well-known law in drug release studies, the Peppas law.

4.2.2 Validation of theoretical model based on experimental results

Gelatin and poly(vinyl alcohol) (GEL-PVA) micro-particles cross-linked with glutaraldehide (GA), for samples prepared by using different amount of cross-linking agent (2%, 6%, 8%, 10% - the sample code indicate the cross-linking amount: for example, GA2 represents a sample with 2% cross-linking amount), loaded with chloramphenicol, were studied [118].

We analyzed these results fitting the experimental data with a Weibull type law, demonstrated in the above paragraph. As a result, we obtained parameters *a* and *b*, the correlation factors and release kinetics fractal dimension for each of the samples that indicates some information on the drug release mechanisms at mesoscopic scale (see Table 4.2).

The first observation is that the correlation coefficient between the experimental curves and Weibull fitted curve are very good, better that than for Peppas curve, this allowing us to affirm that this entire release process can be described better by the Weibull type law instead of Peppas law, showing the wide applicability of a Weibull type law.

4.3 The dispersive approximation of motion in drug release processes

4.3.1 The generalized diffusion equation

In dispersive case, (4.55) take the form of generalized diffusion equation:

$$\frac{\partial Q}{\partial t}+(\boldsymbol{V}-\boldsymbol{U})\cdot\nabla Q+\frac{\sqrt{2}}{3}\boldsymbol{\mathcal{D}}^{3/2}(dt)^{(3/D_F)-1}\nabla^3 Q=0 \qquad (4.77)$$

Assuming that $|V-U|=\sigma\cdot Q$ with $\sigma=constant$ [108], in the one-dimensional case and introducing a dimensionless coordinate system, equation (4.77) take the standard form of the Korteweg de Vries equation

$$\partial_\tau\phi+6\phi\partial_\xi\phi+\partial_{\xi\xi\xi}\phi=0 \qquad (4.80)$$

A stationary solution of equation (4.80) has the expression

$$\phi(\xi,\tau,s)=2a\left(\frac{E(s)}{K(s)}-1\right)+2a\cdot cn^2\left[\frac{\sqrt{a}}{s}\left(\xi-\frac{u}{2}\tau+\xi_0\right);s\right] \qquad (4.83)$$

where *cn* is the Jacobi's elliptic function of s modulus [109], a is an amplitude, ξ_0 is a constant of integration and

$$K(s)=\int_0^{\pi/2}\left(1-s^2\sin^2\varphi\right)^{-1/2}d\varphi, E(s)=\int_0^{\pi/2}\left(1-s^2\sin^2\varphi\right)^{1/2}d\varphi \qquad (4.84)$$

are the complete elliptic integrals [109]. As a result, the drug particle movement is achieved by one-dimensional cnoidal oscillation modes of the concentration field (Figure 4.9).

4.3.2 Experimental results

4.3.2.1 The preparation protocol

Most of the experimental data presented in literature reveals that, in general, the drug release from polymeric matrixes takes place in accordance to a power law in the first 60% of the release curve and/or to exponential Weibull law on the entire drug release curve, reaching an average constant equilibrium value. These results are, generally, for experiments carried out on relatively short time intervals, for which the dominant phenomena are disolution and diffusion, the system exhibiting a „burst effect" due to the high concentration gradient, followed by a linear evolution, on a constant value, corresponding to the equilibrium state with no concentration gradient.

But, some experimental results, caried out on time intervals long enough so that the process to evolve completely, including the polymer degradation stage, shows unsual behaviours, with strong fluctuations.

4.3.2.2 Levofloxacin release kinetics

The release of levofloxacin loaded in the above described micro-particles are plotted in Figure 4.10. The plots are grouped after the variable preparation parameter.

All these experiments were caried out on 28 days, the concentration of the released drug being measured daily, at the same hour. The general characteristic of the above kinetics are the strong variations of concentration in time.

We must point out that if these experiments would have take place at time scales of hours order, the system behaviours would have followed the well-known exponential Weibull law, analyzed in [96]. The corresponding release kinetics can be observed in Figure 4.11.

4.3.3 Validation of theoretical model based on experimental results

In the following, we identify the field Φ with the normalized concentration field of the released drug from micro-particles. In such context, it results that the drug release mechanism is accomplished by means of cnoidal oscillation modes of the normalized concentration field, the parameter *s* representing a measure of the system nonlinearity degree. So, the one-dimensional cnoidal oscillation modes contain as subsequences for $s=0$ the one-dimensional harmonic waves, while for $s\to 0$ the one-dimensional waves packet. These two subsequences define the non-quasi-autonomous regime of the drug release process. For $s=1$, the solution (4.83) becomes a one-dimensional soliton, while for $s\to 1$ the one-dimensional solitons packet results. These last two subsequences imply the quasi-autonomous regime for drug particle release process.

To find the best correlation between the experimental data and the theoretical model, for each sample, we used a planar intersection of the graph in Fig. 4.9, where the two variables are $y=\xi-\tau u/2$ si $x=s$. With these variables equation (4.83) becames:

$$\phi_1(x,y) = 2a\left(\frac{E(x)}{K(x)} - 1\right) + 2a \cdot cn^2\left[\frac{\sqrt{a}}{x}\left(y + \xi_0\right); x\right] \quad (4.85)$$

Thus, in order to find the one-dimensional equation for a planar intersection, perpendicular to plane xOy, we used $y = mx + n$, equation of a linear function, where m and n are two parameters. After substitutions we obtain an one-dimensional function:

$$\phi_2(t,m,n) = \phi_1\left(\frac{t}{\sqrt{m^2+1}}, n + m\frac{t}{\sqrt{m^2+1}}\right) \quad (4.87)$$

It results a good agreement between our experimental data and theoretical model.

CONCLUSIONS

The main original results of this thesis are:

Chapter 2

In this chapter we analyzed vacuum from the Casimir cavity, considered a non-differentiable, Newtonian, 2D non-coherent quantum fluid, by writing the Navier-Stokes equations in scale relativity theory's framework. As a result the following results may be extracted :

i) the (vector) velocity field $\boldsymbol{V}$ and/or the (scalar) density field ρ behave like a wave function on small distances (the same magnitude as the Compton length);

ii) the (vector) velocity field $\boldsymbol{V}$ and/or the (scalar) density field ρ are transported by the motion of the Newtonian fluid with speed $\boldsymbol{U}$, on small distances (the same magnitude as the Compton length).

iii) the entities assimilated to vortex-type objects from the Casimir cavity, initially non-coherent, become coherent due to constraints induced by the presence of walls and generate pressure along the *Ox* and *Oy* axis .

iv) It observe that, in the case, of the Casimir cavity from inside a rectangular enclosure of sides d_1, d, the plates induce constraints along both *Ox* and *Oy* axis, and one can notice that if the two parameters *m* and *n* have close values, the force acting on the Casimir rectangle is always negative and for parameters *m* and *n* very asymmetric the force has negative and positive domains, in agreement with the calculus of regularization using the Abel-Plana formula.

Chapter 3

- a complete and detailed nonlinear dynamics analysis (complete time series, Poincaré sections, complete phase space, Lyapunov exponents, bifurcation diagrams, fractal analysis) for a particle-field (electromagnetic field and static magnetic field) nonlinear interaction are analyzed;
- physical mechanisms (gun, chaotic gun and multi-gun effects) which explain the excited states of the atom (classical analogue of quantum absorption) were proposed;
- the fractal analysis of the trajectories resulting from the particle-field nonlinear interaction shows that these curves posses the property of continuity and non-differentiability, *i.e.* they are fractal curves. In such a context, we were able to apply the scale relativity theory for the study of the stationary states of the atom. Therefore, the trajectory fractalisation represents a natural mechanism of

introducing quantification (real part of the complex speed field "select" stationary orbits, while the imaginary part quantifies energy).

Chapter 4

i) on small time scales, we analyzed the experimental results and it is observed that drug release kinetics type corresponds to a Weibull's law. As a result, we obtained the parameters a and b, coefficients of correlation and fractal dimension of release kinetics for each of the samples, all showing some information on dispensing mechanisms mesoscopic or nano scale;

- Values for the correlation coefficient and the experimental curves for the curve described by Weibull are very good. This allows us to say that the whole release process can be described well by a Weibull law than one type Peppas, which leads to large-scale application of a law type Weibull;

ii) on large time scale, we obtained a new model for the mechanism of drug release from polymer matrices considering that the drug particles motion take place on fractal curves

- This model provides a new alternative for the theoretical study of the drug release process, when all phenomena are present, so the complexity of the system, and implicitly, nonlinearity becomes very high.

- It follows that the normalized concentration field, while during the dependence of normalized and systemic nonlinearity (with parameter s). The best values of correlation factors were found, particular plans to the three-dimensional plot of normalized concentration field, indicates that to each state of the system, at a certain moment, corresponds a specific non-linearity, determined by the intrinsic structure of the system.

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